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# Hopping conduction in electron-irradiated boron-doped silicon

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**Abstract.** Hopping conduction is studied in boron-doped Si at low temperatures. A sufficient degree of damage induced by electron irradiation of 1.5 MeV allows observation of conductance via nearest-neighbour hopping or variable-range hopping. Evidence for electron-electron interaction in the hopping conductance is presented.

## 1. Introduction

At very low temperatures it has been observed that conduction in crystalline Si and Ge (Mott and Davis 1971, Allen and Adkins 1972, Sasaki 1985) takes place via hopping of carriers between localised shallow impurity levels when there is compensation present. Radiation-induced defects can act as compensating minority centres. Conduction can then take place via thermally activated hopping. The main advantage of studying hopping conductance in radiation-damaged samples is that the Fermi level can be pinned in the energy gap over a considerable range of temperature by the presence of induced defects, and complications in the study of hopping conductance arising from the temperature dependence of the activation energy for hopping between localised states can be avoided.

The purpose of this work was to study AC hopping conductance in electron-irradiated boron-doped Si. Our measurements show that when  $kT$  is comparable to the band width, then conduction can take place via hopping of carriers between neighbouring sites. If the band width is further increased by damage induced by irradiation, conduction takes place via variable hopping described by Mott's law  $\sigma \approx \sigma_0 \exp(-b/T^{1/4})$ . The experimental parameters of these conductance mechanisms seem to be explained fairly well by the current theories of hopping conductance.

## 2. Experimental procedure

We irradiated thin slabs of Si crystals with evaporated pure gold electrodes at 10 K with 1.5 MeV electrons and we measured the capacitance  $C$  and the loss  $D$  by bridge measurements. The samples were boron-doped, and were either float-zone crystal, with

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room-temperature carrier concentrations of  $N = 1 \times 10^{13}$  or  $1.4 \times 10^{15} \text{ cm}^{-3}$ , or pulled crystals with  $N = 5 \times 10^{15}$  or  $6.5 \times 10^{16} \text{ cm}^{-3}$ .

As we have explained in a previous publication (Kouimtzi 1986a), measurements of the capacitance and loss factor can be directly related to the real and imaginary parts of the AC hopping conductivity. The real part is given by  $\sigma = 6.395 \times 10^{-9} f' D$  ( $D = \omega CR$ ) for reasonably low  $D$  and frequency  $f'$  in the kHz range. In the lowest-resistivity pulled samples the barrier capacitance changes with temperature. The bulk conductivity can then be evaluated using measurements of the loss factor  $D$  and capacitance against  $T$ . The frequency range was 400 Hz–10 kHz.

### 3. Results

Under low-temperature irradiation at these energies and for low doses the main defects expected to be induced in boron-doped Si are charged vacancies and boron interstitials (Watkins 1975, Watkins and Troxell 1980). Following our previous analysis (Kouimtzi 1986b) the carrier removal rate of the sample was monitored from changes in the space charge of the barriers, assuming a Schottky model. The carrier removal rate was found to be  $0.2 \times 10^{-2} \text{ cm}^{-1}$  in samples with  $N = 1 \times 10^{13} \text{ cm}^{-3}$ ,  $0.3 \text{ cm}^{-1}$  for  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$ ,  $0.38 \text{ cm}^{-1}$  for  $N = 5 \times 10^{15} \text{ cm}^{-3}$  and  $0.5 \text{ cm}^{-1}$  for  $N = 6.5 \times 10^{16} \text{ cm}^{-3}$ . The increase in the carrier removal rate with increasing concentration of boron atoms is consistent with previously reported studies (Watkins 1975, Kouimtzi and Banbury 1981).

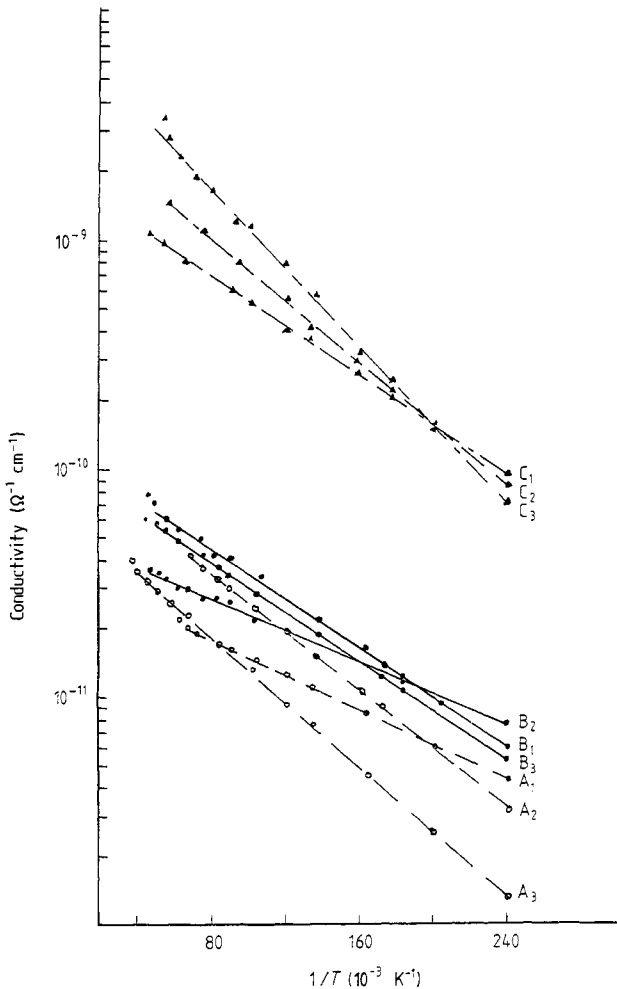
For fairly low doses of irradiation the conductivity at low temperatures exhibits an activation energy of about 1 meV (see figure 1, later). Annealing of the irradiated samples at about 150 K, which is the temperature at which the vacancies become mobile, would cause divacancies and/or vacancy complexes to be introduced into the samples. After such an anneal the temperature dependence of the conductivity at low temperatures could be expressed by Mott's law  $\sigma \propto \exp(-b/T^{1/4})$  for tunnelling-assisted hopping conductance (see figure 4, later). Two different values of  $b$  in two different temperature regimes were observed. The low values of  $b$  were  $b' = 1.7 \text{ K}^{1/4}$  for the highest-resistivity samples with  $N = 1 \times 10^{13} \text{ cm}^{-3}$ ,  $b' = 1.2 \text{ K}^{1/4}$  and  $b' = 1.1 \text{ K}^{1/4}$  for samples with carrier concentrations  $N = 1.4 \times 10^{15}$  or  $5 \times 10^{15} \text{ cm}^{-3}$ , while the corresponding high values were  $b = 4.1$  and  $3.1 \text{ K}^{1/4}$ , respectively.

At higher temperatures activated behaviour was again observed (see figure 5, later). For heavily irradiated samples the value of  $b$  increased considerably, with  $b_3 = 26 \text{ K}^{1/4}$  and  $b_4 = 22 \text{ K}^{1/4}$  (see figure 8, later).

### 4. Discussion

For fairly low doses of irradiation the conductivity at low temperatures exhibits an activation energy of about 1 meV (figure 1), which is of the order of  $kT$  at these low temperatures. We could associate these low energies ( $\mathcal{E}_3$ ) with nearest-neighbour hopping: they would then represent the separation between the Fermi level and the centre of the band produced by the energy perturbation due to the ionised acceptors and/or defects. Miller and Abrahams (1960), considering the movement of the Fermi level with increasing compensation factor ( $K$ ), found that

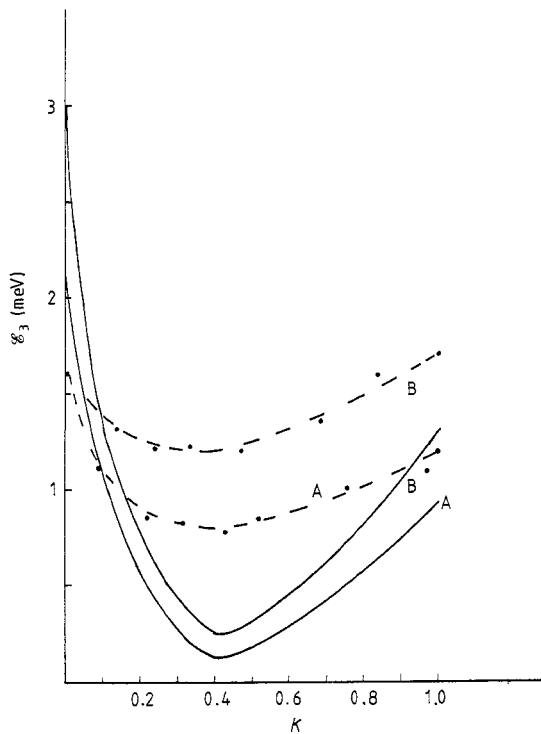
$$\mathcal{E}_3 = (e^2/\kappa)(4\pi N_{\text{maj}}/3)^{1/3}(1 - 1.35K^{1/3}).$$



**Figure 1.** The conductivity of electron-irradiated p-type Si at 1.5 MeV as a function of reciprocal temperature ( $f = 1$  kHz). The curves are as follows.

- A<sub>1</sub>:  $N = 1 \times 10^{13} \text{ cm}^{-3}$  after irradiation with  $\Phi = 1 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 0.9 \text{ meV}$ ).
- A<sub>2</sub>:  $N = 1 \times 10^{13} \text{ cm}^{-3}$  after irradiation with  $\Phi = 2 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 1.3 \text{ meV}$ ).
- A<sub>3</sub>:  $N = 1 \times 10^{13} \text{ cm}^{-3}$  after irradiation with  $\Phi = 2.3 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 1.5 \text{ meV}$ ).
- B<sub>1</sub>:  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 0.8 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 1.1 \text{ meV}$ ).
- B<sub>2</sub>:  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 2 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 0.8 \text{ meV}$ ).
- B<sub>3</sub>:  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 4.5 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 1.1 \text{ meV}$ ).
- C<sub>1</sub>:  $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 6.2 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 1.1 \text{ meV}$ ).
- C<sub>2</sub>:  $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 9 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 1.35 \text{ meV}$ ).
- C<sub>3</sub>:  $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 1.2 \times 10^{16} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 1.7 \text{ meV}$ ).

That is, a minimum in the activation energies should be observed at  $K = 0.4$ . More elaborate calculations have been carried out by Shklovskii (1973) and Shklovskii and Efros (1976) yielding activation energies lower by a factor of 0.8 and a minimum at  $K = 0.5$ . In figure 2 we show the activation energies  $\mathcal{E}_3$  calculated from the formula of Miller and Abrahams (1960) as well as the experimentally observed ones, which do not exhibit very steep variations with increasing compensation factor and are higher than the ones predicted theoretically by Miller and Abrahams or Shklovskii.

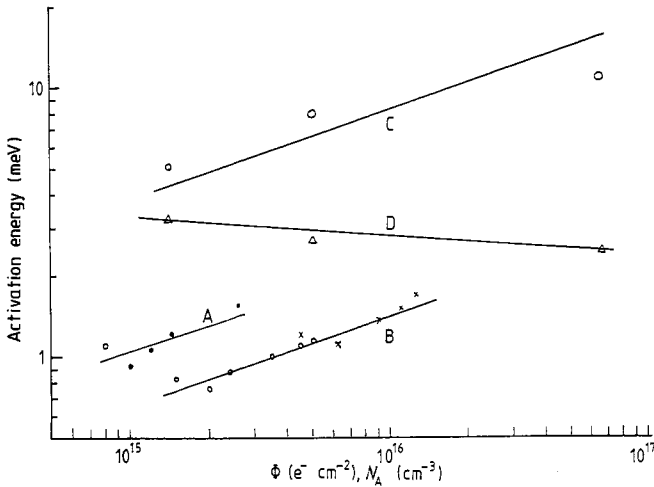


**Figure 2.** The activation energy  $\mathcal{E}_3$  in meV as a function of compensation. The full curves represent the values calculated using the equation of Miller and Abrahams (1960) and the broken curves represent the experimental values. Curve A:  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$ ; curve B:  $N = 5 \times 10^{15} \text{ cm}^{-3}$ .

This discrepancy should reflect the band broadening that occurs in electron-irradiated Si, as has been observed in IR studies by Kalma (1973), and the fact that the Fermi level could be pinned in the gap by a defect level. It would then be expected that the activation energy for nearest-neighbour hopping will vary as the cube root of the irradiation dose. In figure 3 we show the variation of the observed activation energies versus the dose ( $\Phi$ ) of irradiation, which is very close to the line that indicates the  $\Phi^{1/3}$  variation. Thus we are led to attribute the above activation energies to nearest-neighbour hopping of carriers.

This change in the activation energy will result in a rather weak dependence of the conductivity on the dose of irradiation. The curve of the conductivity versus the dose will mainly be determined by the number of carriers on the doping atoms that can hop because of the pre-exponential factor in the conductivity ( $\sigma \propto N \exp(\mathcal{E}_3/kT)$ ) (Mott and Davis (1971)), as has been previously assumed in experiments performed by McKeighen and Koehler (1971), Kouimtzi and Banbury (1981, 1982), and Bains and Banbury (1987). The dependence of the DOS  $N$  on the dose  $\Phi$  will be of the form  $(N_A - N_{\text{def}})N_{\text{def}}$  (Kouimtzi and Banbury 1981) giving a maximum at  $K \approx 0.5$  as has already been observed in a series of experiments (Kouimtzi and Banbury 1981, 1982, Kouimtzi 1987).

The band width could be further increased either by increasing the irradiation dose  $\Phi$  or by annealing the samples at 150 K because of the formation of complex defects (Watkins 1975, Harris and Watkins 1985). After such an anneal the variation of  $\sigma$



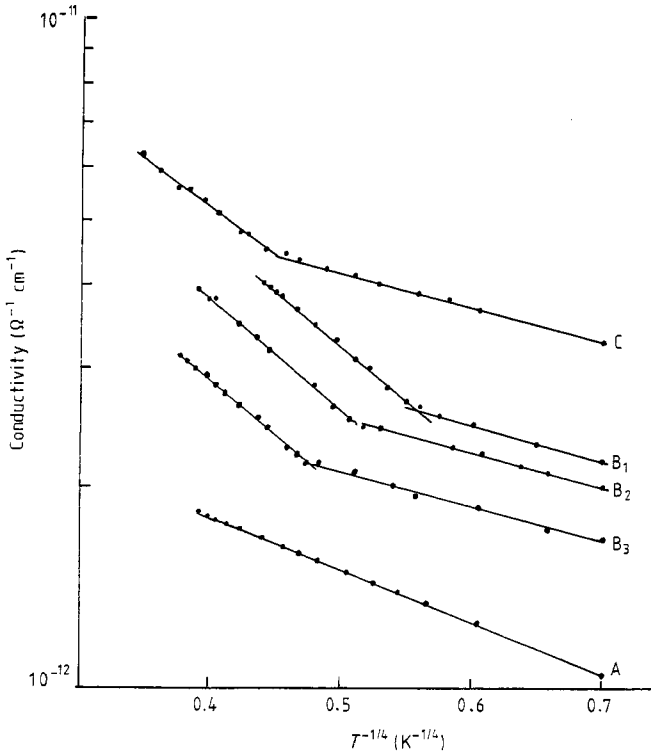
**Figure 3.** Curves A and B: the activation energy  $\mathcal{E}_3$  in electron-irradiated boron-doped Si after irradiation plotted against electron dose ( $f = 1$  kHz). The dots show the experimental points for samples with  $N = 1 \times 10^{13} \text{ cm}^{-3}$ ; the circles the experimental points for samples with  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  and the crosses the experimental points for samples with  $N = 5 \times 10^{15} \text{ cm}^{-3}$ .

Curves C and D: the activation energies  $\mathcal{E}_3$  and  $\mathcal{E}_2$  respectively in electron-irradiated boron-doped Si after irradiation and annealing as a function of the concentration of doping atoms. (The compensation factor immediately after irradiation was estimated to be  $K \approx 0.6\text{--}0.7$ ).

with  $T$  could be expressed by Mott's law  $\sigma \propto \exp(-b/T^{1/4})$  for variable-range hopping (figure 4).

The high values of  $b$  observed by us vary approximately as  $N_A^{-1/4}$  as is expected from Mott's law,  $b \propto (N(E_F))^{-1/4}$ , since the DOS  $N(E)$  should be proportional to the concentration of hopping centres—that is, to the density of acceptors  $N_A$ . For tunnelling-assisted hopping, Ambegaokar (1976) stated that the values of  $b$  should fulfil the criterion  $b^4 > 200 U/k$ . Assuming that the binding energy  $U$  is only the Coulomb potential of the charged defects ( $U \approx 2 \text{ meV}$ ), the value of  $b$  that fulfils such an assumption is  $b = 8 \text{ K}^{1/4}$ , which is higher than the values of  $b$  we observed and consistent with the observation of Morigaki (1980) that the experimental values of  $b$  are lower than those assumed by Ambegaokar. According to Morigaki (1980) the reason for this discrepancy is not yet clear, but we believe that in the irradiated p-type Si, the existence of neutral defects (Watkins and Troxell 1980) randomly distributed in space can cause localisation of carriers, but the binding energy of the localised carriers would be lower than that of the charged defects.

From the measured values of  $b$  we can estimate the tunnelling parameter  $\alpha^{-1}$  since  $b = 2.1 (\alpha^3/kN(E))^{1/4}$  (Mott and Davis 1971). Assuming that the band width will be determined by the energy perturbation due to a singly charged centre at a distance that we state to be the inter-donor spacing, we can make an estimate of the DOS. This gives a state density of  $6 \times 10^{17} \text{ cm}^{-3} \text{ eV}^{-1}$  and  $2 \times 10^{17} \text{ cm}^{-3} \text{ eV}^{-1}$  for samples with acceptor concentrations  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  and  $N = 5 \times 10^{15} \text{ cm}^{-3}$  respectively. The tunnelling parameter  $\alpha^{-1}$  is then estimated to be about  $1000 \text{ \AA}$ . We can proceed further by



**Figure 4.** The conductivity (log scale) of electron-irradiated boron-doped Si as a function of  $T^{-1/4}$  at 1 kHz. The curves are as follows.

- A:  $N = 1 \times 10^{13} \text{ cm}^{-3}$  after irradiation with  $\Phi = 3.2 \times 10^{16} \text{ e}^- \text{ cm}^{-2}$  ( $b' = 1.7 \text{ K}^{1/4}$ ).  
 B:  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  after irradiation and annealing at about 150 K.  
 B<sub>1</sub>:  $\Phi = 2 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $b' = 1.2 \text{ K}^{1/4}$ ,  $b = 4 \text{ K}^{1/4}$ ).  
 B<sub>2</sub>:  $\Phi = 3.5 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $b' = 1.2 \text{ K}^{1/4}$ ,  $b = 4.1 \text{ K}^{1/4}$ ).  
 B<sub>3</sub>:  $\Phi = 4.5 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $b' = 1.2 \text{ K}^{1/4}$ ,  $b = 4.1 \text{ K}^{1/4}$ ).  
 C:  $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation and annealing at about 150 K with  $\Phi = 9 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$ ,  $b' = 1.1 \text{ K}^{1/4}$ ,  $b = 3.1 \text{ K}^{1/4}$ .

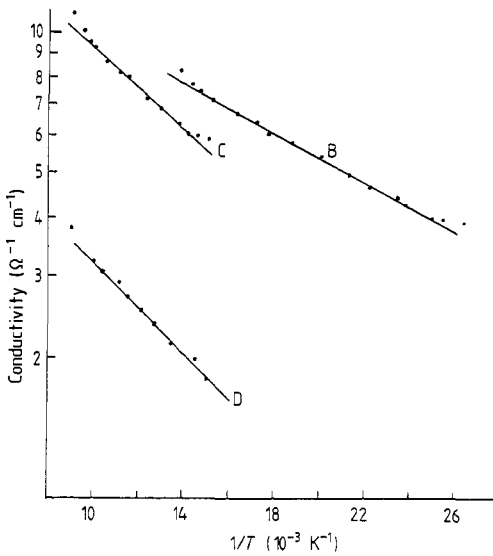
estimating the distance  $R$  for which tunnelling occurs

$$R = [(8\pi/9)\alpha kTN(E)_F]^{-1/4}$$

which is found to be larger than the donor spacing; thus hopping is not to the nearest neighbour.

As we have already mentioned, at very low temperatures lower values of  $b'$  were observed. Similar low values of  $b$  have been reported in the literature and attributed to electron–electron interactions (Morigaki 1980, Knotek and Pollak 1974). It is generally agreed that such interactions produce a Coulomb gap in the impurity band. Conduction involving single-electron hops occurs only if  $\mathcal{E}_g < kT$ . For  $kT < \mathcal{E}_g$ , Mott and Davies (1980) conclude that conduction would occur by many-electron hops (variable-number hopping) and the  $T^{-1/4}$  law should still be valid but with a lower pre-exponential factor, as in our experimental observations of  $\log \sigma$  versus  $T^{-1/4}$  in the temperature regime with the low values of  $b$ .

According to Pollak (1980), variable-number hopping takes place at temperatures  $\theta \leq \mathcal{E}_g \alpha^{-1}/2rk$ . The Coulomb gap has been calculated by several authors (Pollak 1980,



**Figure 5.** The conductivity (log scale) of electron-irradiated boron-doped Si as a function of reciprocal temperature. The curves are as follows.

B:  $\sigma_{AC} \times 10^{12} (\Omega^{-1} \text{ cm}^{-1})$   $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  after irradiation and annealing at about 150 K with  $\Phi = 3.5 \times 10^{15} \text{ e}^{-} \text{ cm}^2$  ( $\mathcal{E}_3 = 5.2 \text{ meV}$ ).

C:  $\sigma_{AC} \times 10^{11} (\Omega^{-1} \text{ cm}^{-1})$   $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation and annealing at about 150 K with  $\Phi = 9 \times 10^{15} \text{ e}^{-} \text{ cm}^2$  ( $\mathcal{E}_3 = 8 \text{ meV}$ ).

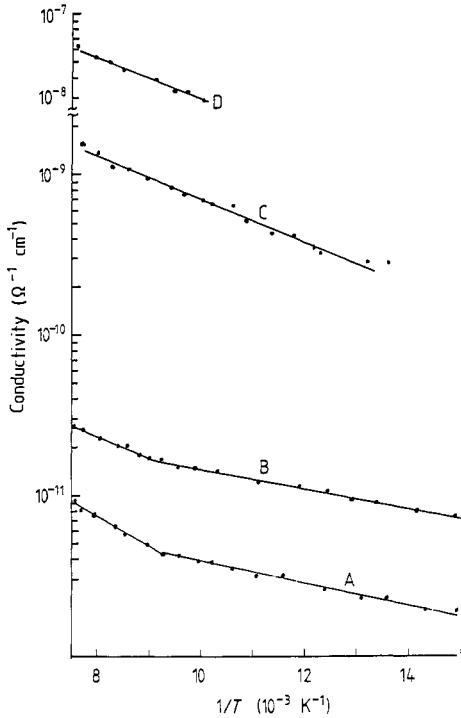
D:  $\sigma_{AC} \times 10^8 (\Omega^{-1} \text{ cm}^{-1})$   $N = 6.5 \times 10^{16} \text{ cm}^{-3}$  after irradiation with  $\Phi = 5 \times 10^{17} \text{ e}^{-} \text{ cm}^{-2}$  ( $\mathcal{E}_3 = 10 \text{ meV}$ ).

Pollak and Ortuno 1982, Ortuno and Pollak 1985). In the simplest and most accurate model of Pollak (1980), the Coulomb gap is estimated to be  $\mathcal{E}_g = \mathcal{E}_C (\mathcal{E}_C / \Delta)^{1/4}$ , where  $\mathcal{E}_C$  and  $\Delta$  are the band widths of the majority (acceptor) and minority (defect) centres respectively, which could be expressed as  $\mathcal{E}_g = \mathcal{E}_C (N_{\text{def}} / N_A)^{1/6}$ . It is thus expected that  $\Theta$  will increase with a progressive degree of damage due to the band broadening of the impurity band, and with increasing doping concentration. This is consistent with our experimental observations (figure 4). According to the quoted carrier removal rate,  $\mathcal{E}_g$  should be about 2 meV, in accordance with Mott's simple requirement ( $\mathcal{E}_g < kT$ ) and  $\Theta \approx 20 \text{ K}$ , consistent with our experimental observations.

At a temperature  $T = 3\Theta$  (Knotek and Pollak 1974) nearest-neighbour hopping is expected to take over from variable-range hopping. Activated conduction was observed at higher temperatures (figure 5). These activation energies vary approximately as the cube root of the acceptor concentration (figure 3, curve C) for reasons we have already stated. They are higher than the ones assumed by Miller and Abrahams (1960), indicating that the induced complex defects in the annealed samples have caused further broadening of the impurity band.

At higher temperatures we observed activation energies (figure 6) of about 0.03 eV. Similar activation energies have been attributed by Bains and Banbury (1987) to boron-substitutional-vacancy complexes. We feel that our activation energies cannot be attributed to such defects because they decrease with increasing acceptor concentration: they should be attributed to band conduction since their decrease should reflect the band broadening. The ionisation energies of 0.013 and 0.012 eV observed in the samples with





**Figure 6.** The conductivity of electron-irradiated boron-doped Si as a function of reciprocal temperature ( $f = 1$  kHz). The curves are as follows.

A:  $N = 1 \times 10^{13} \text{ cm}^{-3}$  after irradiation with  $\Phi = 3.2 \times 10^{16} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_2 = 0.013 \text{ eV}$ ,  $\mathcal{E}_1 = 0.035 \text{ eV}$ ).

B;  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  after irradiation and annealing at about 150 K with  $\Phi = 3.5 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_2 = 0.012 \text{ eV}$ ,  $\mathcal{E}_1 = 0.032 \text{ eV}$ ).

C:  $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 2.5 \times 10^{17} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_1 = 0.027 \text{ eV}$ ).

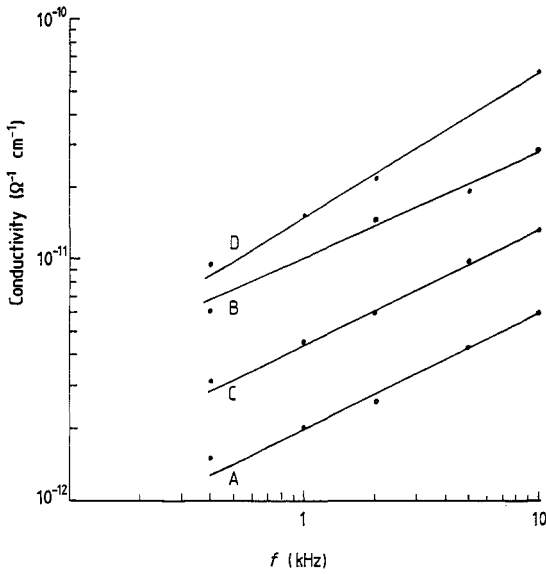
D:  $N = 6.5 \times 10^{16} \text{ cm}^{-3}$  after irradiation with  $\Phi = 5 \times 10^{17} \text{ e}^- \text{ cm}^{-2}$  ( $\mathcal{E}_1 = 0.025 \text{ eV}$ ).

$N = 1 \times 10^{13} \text{ cm}^{-3}$  and  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$ , respectively, at temperatures lower than the ones where band conduction occurs, should be attributed to conduction in the  $\mathcal{E}_2$  band. Conduction in the  $\mathcal{E}_2$  band might be by hopping or a band mechanism. It was interesting to observe that the frequency dependence of the conductivity was sublinear (figure 7, curves A and B), which is evidence (Pollak and Geballe 1960, Pollak and Hunt 1985) that conduction occurs via hopping.

According to Mycielski (1961), the activation energy  $\mathcal{E}_2$  is  $\mathcal{E}_1 - 3(e^2/\kappa r)$ . The term in parentheses represents three times the band width of the impurity band, that is  $\mathcal{E}_3$ . For samples with  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$ , we have observed, as mentioned previously, that  $\mathcal{E}_3 = 5.2 \text{ meV}$ . Mycielski's calculation would then yield  $\mathcal{E}_2 = 0.015 \text{ eV}$ , which is close to the experimental values we observed.

## 5. Effects on heavily irradiated specimens

It was interesting to observe that in heavily irradiated pulled samples the value of  $b$  increased considerably (figure 8), with  $b = 26 \text{ K}^{1/4}$  and  $b = 22 \text{ K}^{1/4}$  and  $N = 5 \times 10^{15} \text{ cm}^{-3}$



**Figure 7.** The frequency dependence of the conductivity of electron-irradiated boron-doped Si. The curves are as follows.

A:  $N = 1 \times 10^{13} \text{ cm}^{-3}$  after irradiation at 1.5 MeV with  $\Phi = 3.2 \times 10^{16} \text{ e}^- \text{ cm}^{-2}$  ( $\sigma \propto \omega^{0.45}$ ,  $T = 70 \text{ K}$ ).

B:  $N = 1.4 \times 10^{15} \text{ cm}^{-3}$  after irradiation at 1.5 MeV and annealing at about 150 K.  $\Phi = 3.5 \times 10^{15} \text{ e}^- \text{ cm}^{-2}$  ( $\sigma \propto \omega^{0.4}$ ,  $T = 75 \text{ K}$ ).

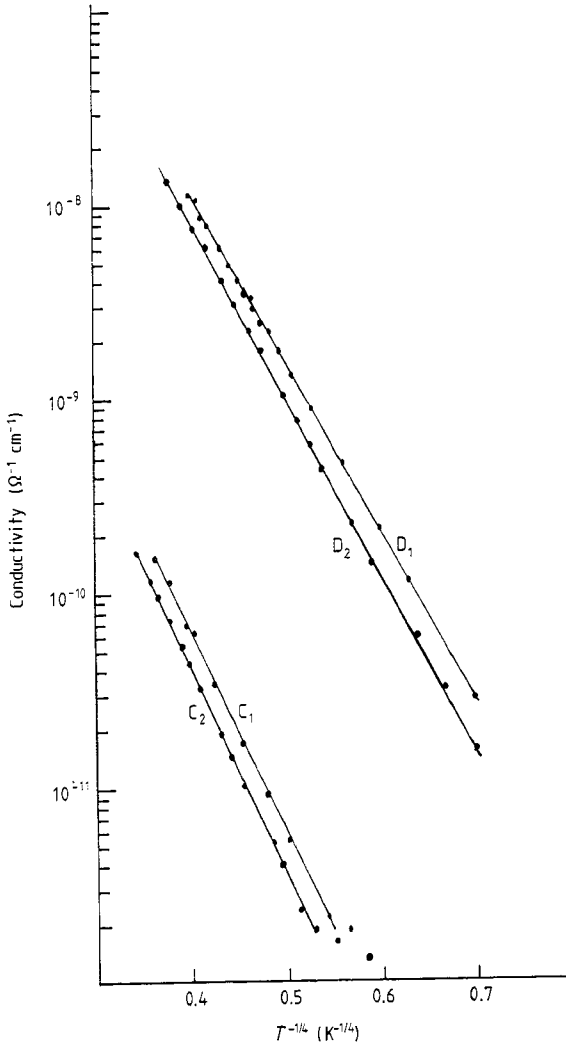
C:  $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation at 2 MeV with  $\Phi = 2.5 \times 10^{17} \text{ e}^- \text{ cm}^{-2}$  ( $\sigma \propto \omega^{0.45}$ ,  $T = 18 \text{ K}$ ).

D:  $N = 6.5 \times 10^{16} \text{ cm}^{-3}$  after irradiation at 2 MeV with  $\Phi = 1 \times 10^{18} \text{ e}^- \text{ cm}^{-2}$  ( $\sigma \propto \omega^{0.6}$ ,  $T = 4.2 \text{ K}$ ).

and  $N = 6.5 \times 10^{16} \text{ cm}^{-3}$  respectively. The observed values of  $b$  seem to be independent of acceptor concentration and comparable with relative values for amorphous Si quoted in the literature (Kashimoto and Morigaki 1979, Morigaki 1980).

There are no available data for the DOS in electron-irradiated Si. We can make a reasonable estimate by assuming the band width to be of the order of the activation energy for nearest-neighbour hopping (about 10 meV) (figure 5). The DOS thus estimated is close to that for the valence band in pure Si, and the tunnelling parameter is then found to be  $\alpha^{-1} = 80 \text{ \AA}$  and  $\alpha^{-1} = 25 \text{ \AA}$  for samples with acceptor concentration  $N_3$  and  $N_4$  respectively. Similar values have been quoted by Sasaki (1985) for neutron-irradiated Si, as well as by Morigaki (1980) and Chik and Koon (1986) for amorphous Si.

This great change in the value of  $\alpha$  cannot all be attributed to the Coulomb potential experienced by a tunnelling electron, because this would mean that the concentration of charged defects would have increased by three orders of magnitude, which is impossible according to the carrier removal rate we observed and the ones mentioned so far (Watkins 1975). It was first pointed out by Brower (1970) that multi-vacancy-oxygen clusters are induced in heavily irradiated, mainly pulled, Si. It is thus likely that regions of high defect concentrations exist in our heavily irradiated pulled samples. In these disordered regions, localisation would be strong giving the quoted tunnelling parameter. These multi-vacancy-oxygen centres are asymmetrical and it could be argued that each



**Figure 8.** The conductivity of heavily electron-irradiated boron-doped Si at 2 MeV as a function of  $T^{-1/4}$  ( $f = 1$  kHz). The curves are as follows.

C<sub>1</sub>:  $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 2.5 \times 10^{17} \text{ e}^- \text{ cm}^{-2}$  ( $b = 25 \text{ K}^{1/4}$ ).

C<sub>2</sub>:  $N = 5 \times 10^{15} \text{ cm}^{-3}$  after irradiation with  $\Phi = 8 \times 10^{17} \text{ e}^- \text{ cm}^{-2}$  ( $b = 26 \text{ K}^{1/4}$ ).

D<sub>1</sub>:  $N = 6.5 \times 10^{16} \text{ cm}^{-3}$  after irradiation with  $\Phi = 5 \times 10^{17} \text{ e}^- \text{ cm}^{-2}$  ( $b = 21.5 \text{ K}^{1/4}$ ).

D<sub>2</sub>:  $N = 6.5 \times 10^{16} \text{ cm}^{-3}$  after irradiation with  $\Phi = 1 \times 10^{18} \text{ e}^- \text{ cm}^{-2}$  ( $b = 22 \text{ K}^{1/4}$ ).

region of high defect concentration has relatively extended wings that could overlap to provide the conductivity path. Conduction would then occur via touching of disordered regions. The observed value of  $b$  would then be independent of the acceptor concentration, being characteristic of the individual disordered regions (Coates and Mitchel 1975). This argument could then explain the differences between the values of  $b$  reported so far for amorphous and crystalline semiconductors.

The observed frequency dependence is sublinear, of the form  $\omega^s$ , with  $s < 1$  (figure 7, curves C and D) and it provides evidence in favour of such a percolation path. According to Butcher (1980), for values of  $an_s^{-1/3}$  of 7.5 and 10 (where  $n_s$  is the density

of sites) for samples with an acceptor concentration of  $N = 5 \times 10^{15} \text{ cm}^{-3}$  or  $N = 6.5 \times 10^{16} \text{ cm}^{-3}$  respectively, the exponent  $s$  should have a value close to 0.4 as we have observed. The low values of the exponent  $s$  are consistent with previous experiments on the AC conductivity in irradiated semiconductors (InSb and Ge). It has been observed (Kouimtzi 1983, 1986a) that the exponent  $s$  decreases with progressive irradiation from the values that Pollak and Geballe (1961) deduced, i.e.  $s \approx 1$ . The decrease reflects the dependence of  $s$  on the tunnelling parameter in accordance with the percolation theory of Butcher (1980), who extended the theory of Pollak and Geballe (1961). The gradual decrease in  $s$  should reflect the fact that the hopping conductivity gradually becomes independent of frequency as the number of empty sites for electrons to hop to increases.

## 6. Conclusion

The temperature dependence of hopping conductance in electron-irradiated Si has been considered. It has been shown that the temperature dependence of variable-range hopping obeyed the Mott  $T^{-1/4}$  law. The activation energy for nearest-neighbour hopping increased as the concentration of the induced defects increased due to band broadening. The activation energy for conduction in the  $\mathcal{E}_2$  band or in the conduction band decreased due to the same effect. It has been argued that heavy irradiation caused the existence of regions of high defect concentrations.

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